

IMAGE PROCESSING WORKSHOP

Companion eBook

DEMYSTIFYING RECONSTRUCTION USING IMAGEJ

Presenter: Aya Takase

You'll learn:

When you run a CT scan, you always run a reconstruction calculation to convert the 2D projections to 3D volume data. Have you ever wondered how reconstruction works?

In this workshop, you will learn how reconstruction works and become more familiar with terms such as Radon transformation and sinograms.

You can also download the open-source image processing program [ImageJ](#) and run Radon transform and reconstruction calculations to have hands-on experience and deepen your understanding of reconstruction.

Here is the [recording of the workshop](#).

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PRESENTERS



*Presenter: Aya Takase
Director of X-ray Imaging*

Aya holds a PhD in engineering from Osaka University and a MA in physics from Tokyo University of Science and has been with Rigaku for 24 years. She started in the X-ray Diffraction Application Lab and transitioned to X-ray Imaging in 2017. Her goal: Help non-expert X-ray users achieve expert results with less time and effort. She has worked on many projects designing automated and user-friendly X-ray instruments and analysis software. She is very passionate about helping people learn more about X-rays and working with X-ray users to solve their specific problems. [in](#)



*Co-presenter: Angela Criswell
Senior Scientist*

Angela holds a PhD from Rice University and has been with Rigaku for 20 years. She started in the Macromolecular Crystallography Applications lab focusing on X-ray techniques to study structural biology. She has gained expertise in a number of X-ray methods in her tenure at Rigaku, including small angle X-ray scattering and X-ray computed tomography. Angela likes working with customers to find the best fit for their samples while addressing their specific experimental questions. [in](#)



*Host: Tom Concolino
Southeast Regional Account
Manager*

Tom holds a PhD in Chemistry from Mississippi State University and has been with Rigaku for 21 years. He started out in the Small Molecule Crystallography Applications Lab before transitioning to the sales team in 2002. He has been on the front lines helping clients save on time, cost, and effort while pushing forward to support the never-ending need to innovate and explore new materials and structures. From academia to mining to pharmaceutical research, Tom has learned the value of bringing a fresh perspective to each customer application while utilizing his vast experience to collaborate on the best fit solution for each and every customer. [in](#)

What is reconstruction?

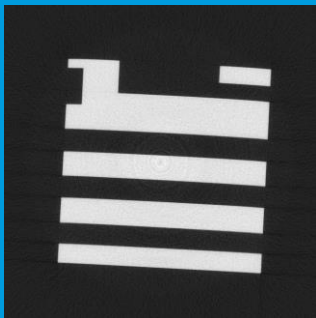
When we run a CT ([computed tomography](#)) scan, we collect radiographs or 2D projections. All portions of the sample in the X-ray beam path get projected on top of each other. How do we convert them into a 3D map of the object or a collection of tomographic cross-sections? This process of restoring the original higher-order dimensions from lower-order observable values is called reconstruction.

Reconstruction is arguably the most important concept of X-ray CT. Because most [artifacts](#) are generated in this process, it is important to understand reconstruction to prevent or reduce these artifacts. However, reconstruction is also the most confusing part of image processing involved in CT.

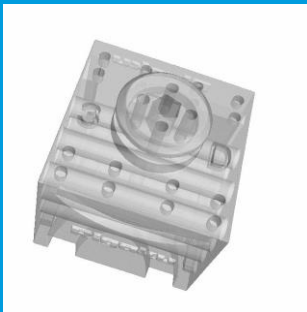
EXAMPLE



Projections



Cross-sections



3D rendering

Figure 1: Projections, cross-sections, and 3D rendering.

Reconstruction

Rendering

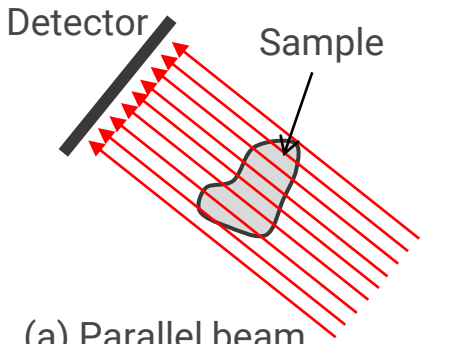
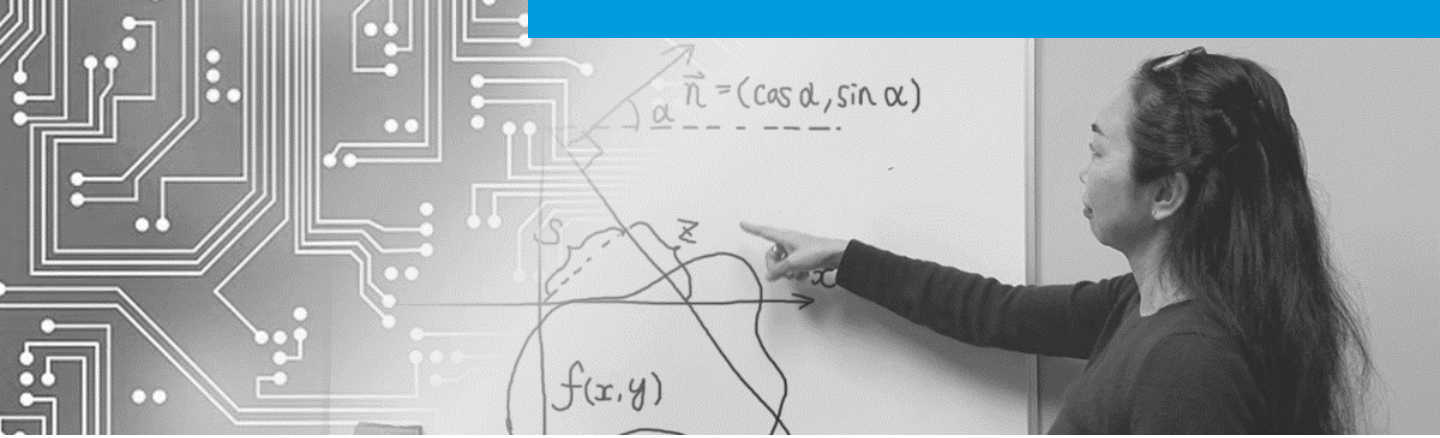
The first thing some people are confused about is which part of their everyday data processing is reconstruction.

During the data collection, we collect 2D projections observed on the detector. (Note: These can be 1D profiles when a line detector is used.) Converting the 2D projections into 3D data or a set of cross-sections is the reconstruction process.

We sometimes see a CT scan represented in a 3D rendered view. This is merely a different representation of a set of cross-sections.

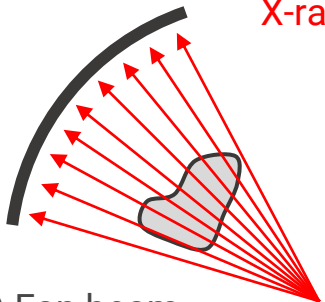
Example

Figure 1 shows different stages and representations of X-ray image data: projections, cross-sections, and 3D rendering, using a 3D printed plastic object as an example.

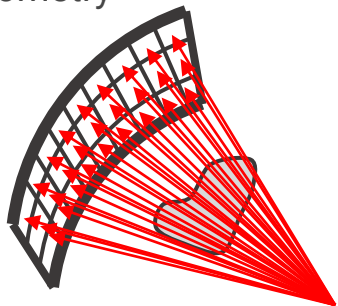


(a) Parallel beam geometry

Incident X-rays



(b) Fan beam geometry



(c) Cone beam geometry

Figure 2: Widely used X-ray CT geometries

X-ray CT Geometries

As you will see later, we attempt to “back-calculate” the absorption coefficient distribution in the sample based on the observed projections. To do this, we need to model the X-ray CT geometry.

Figure 3 shows three commonly used X-ray CT geometries. To keep it simple, we will focus on the parallel beam geometry here. You can extend the theory we will review here to other geometries.

If you are interested, this video shows how the filtered back projection calculation is extended from the parallel beam to the fan beam geometry: [Intro to Digital Image Processing by Rich Radke - # 19 Fan beam reconstruction.](#)

The concept of reconstruction was first presented by Johann Radon in 1917. [The word “tomography” didn’t even exist at that time](#), but his theory proved that CT reconstruction was possible. It was later demonstrated by experiments, and commercial medical CT scanners became available in 1971. For the development of commercial CT scanners, Sir Godfrey N. Hounsfield and Allan M. Cormack, were awarded [the Nobel Prize in Physiology or Medicine in 1979](#).

Today, there are a variety of reconstruction techniques. Here is a non-exhaustive list:

- Algebraic Reconstruction Technique
- Iterative Reconstruction
- Filtered Back Projection
- Convolution Back Projection
- Deep Learning Reconstruction

RECONSTRUCTION TECHNIQUES

Algebraic Reconstruction Technique

Let’s review the most basic technique, Algebraic Reconstruction Technique (ART) first. This technique, as its name indicates, algebraically reconstructs the original absorption coefficient distribution.

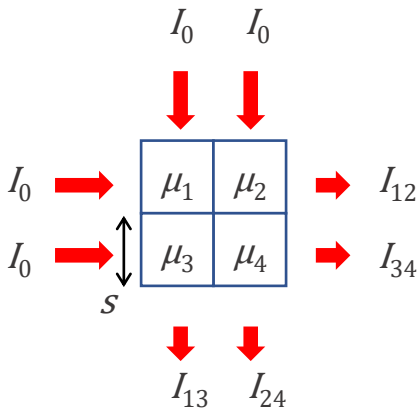


Figure 3: 2 by 2-pixel experiment

To keep it simple, we consider a 2 by 2-pixel object. The pixel thickness is s , and each pixel has a unique linear absorption coefficient μ_1, μ_2, μ_3 , and μ_4 as shown in Figure 3. We put an X-ray beam with intensity I_0 through the sample in the horizontal direction and observe the transmitted X-ray intensities I_{12} and I_{34} . Then we change the X-ray beam direction and repeat the same experiment in the vertical direction to obtain I_{13} and I_{24} .

The relationship between the transmitted and incident X-ray intensities follows the Beer-Lambert law:

$$I = I_0 e^{-\mu s}$$

where μ and s are the linear absorption coefficient and thickness of the object I_0 goes through, respectively.

Using this relationship, we can describe the 2 by 2-pixel experiment as follows:

$$I_{12} = I_0 \exp\{-(\mu_1 + \mu_2)s\}$$

$$I_{34} = I_0 \exp\{-(\mu_3 + \mu_4)s\}$$

$$I_{13} = I_0 \exp\{-(\mu_1 + \mu_3)s\}$$

$$I_{24} = I_0 \exp\{-(\mu_2 + \mu_4)s\}$$

BEER-LAMBERT LAW

Here, we have four equations and four unknown parameters to figure out to know the exact absorption coefficient distribution inside of this 2 by 2-pixel object. Therefore, we can solve these relational equations to obtain μ_1 , μ_2 , μ_3 , and μ_4 .

This is called the Algebraic Reconstruction Technique. Because this is a direct calculation of the absorption coefficient distribution, it suffers from fewer artifacts compared to other techniques. It tolerates a lack of projections better than other techniques. For those reasons, it is used for medical CT reconstruction in some cases.

Further Reading

Radon, J. *On the determination of functions from their integral values along certain manifolds*, IEEE Trans. Med. Imaging, 5 (4). (English translation published in 1986)

<https://doi.org/10.1109/TMI.1986.4307775>

RADON'S ORIGINAL
PAPER (ENGLISH
TRANSLATION)

2 What is projection data?

We have been using the term “projection.” Before moving on to another reconstruction technique, let’s define what projection data is. This is an important concept to understand before getting into the next technique, Filtered Back Projection (FBP).

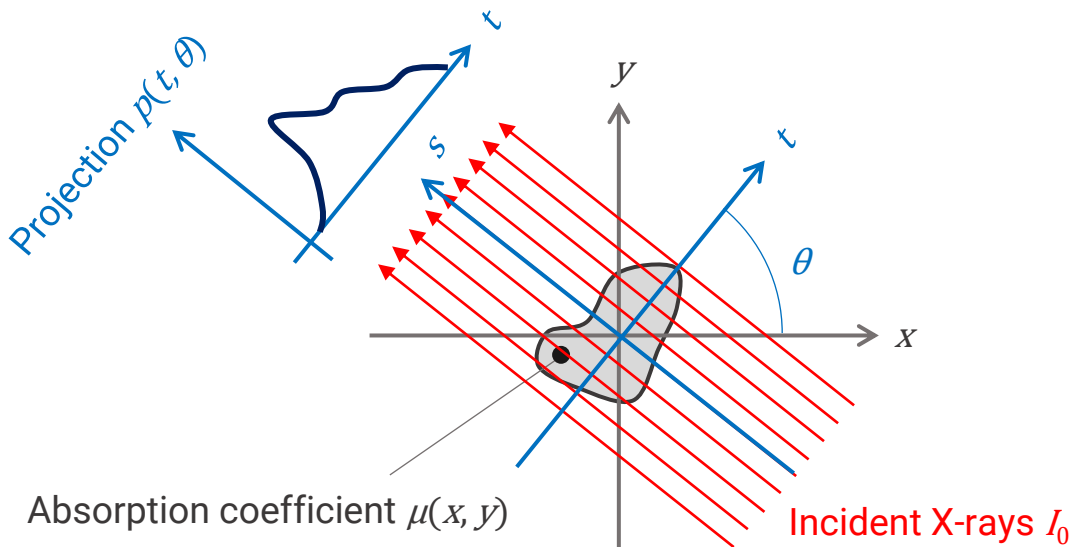
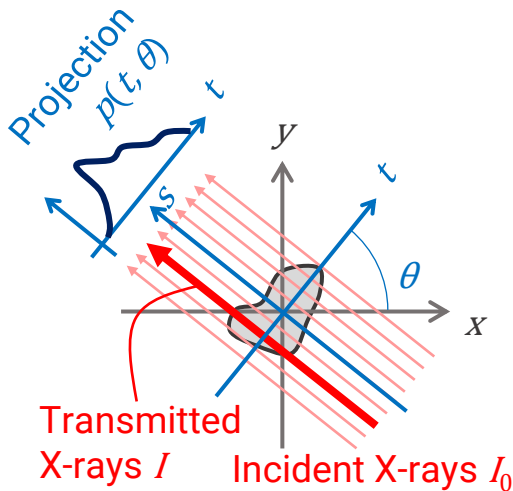


Figure 4: Experimental setup. The scanned object is $\mu(x, y)$ and generates a projection $p(t, \theta)$ at angle θ when X-rays I_0 travel through it.



Projection $p(t, \theta)$

A projection is what we observe on the detector after X-rays transmit through an object. Let's look at one X-ray beam, I_0 , as shown in Figure 5. The transmitted X-ray intensity I follows Beer-Lambert law:

$$I = I_0 e^{-\mu s}$$

Figure 5: Experimental setup

The linear absorption coefficient μ is a function of the location (x, y) , as shown in Figure 4, therefore:

$$\frac{I}{I_0} = \exp \left[- \int_{-\infty}^{\infty} \mu(t \cos \theta - s \sin \theta, t \sin \theta + s \cos \theta) ds \right]$$

Using this equation, we can define the projection $p(t, \theta)$ as absorption integrated through the thickness:

$$p(t, \theta) = -\log \frac{I(t, \theta)}{I_0} = \int_{-\infty}^{\infty} \mu(t \cos \theta - s \sin \theta, t \sin \theta + s \cos \theta) ds \quad (*)$$

Radon Transformation

When we run a CT scan, we collect a set of projections $p(t, \theta)$ for a wide range of θ , normally $0 - 180$ degrees for the parallel beam geometry, $0 - 360$ degrees for the cone beam geometry. As we collect the data, we are converting $\mu(x, y)$ into $p(t, \theta)$.

$$\mu(x, y) \rightarrow p(t, \theta)$$

This conversion, expressed in equation (*) on page 11, is called Radon transformation or forward projection.

Sinogram

Let's take a look at an example of Radon transformation using the cross-section shown in Figure 6. You see a high-density (bright) egg-shaped object in the air (black). The "egg" has a low-density (gray) yolk-like circle. The absorption coefficient of each location is $\mu(x, y)$.

You can convert this cross-section $\mu(x, y)$ into $p(t, \theta)$ using equation (*) and obtain the sinogram, also shown in Figure 6.

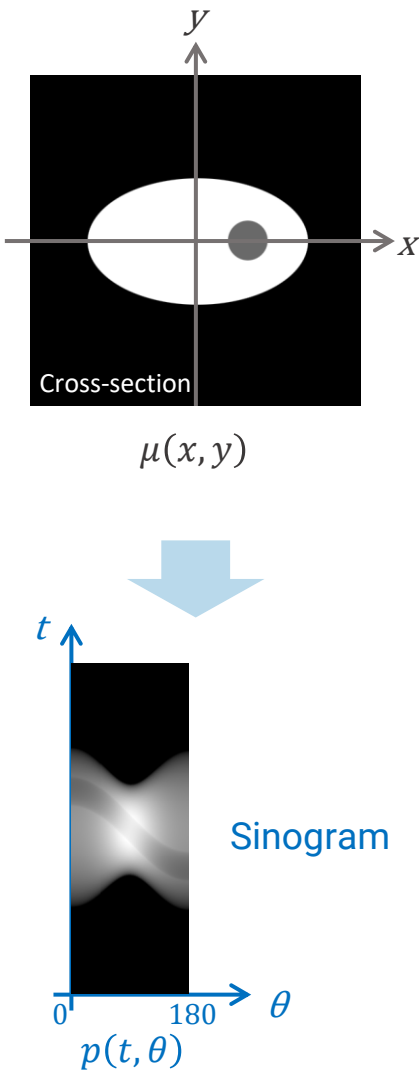
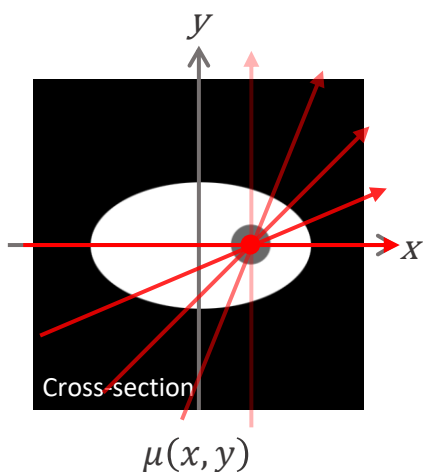
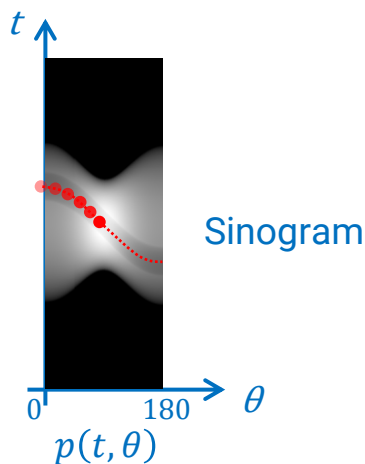


Figure 6: Original absorption coefficient distribution $\mu(x, y)$ shown in a cross-section (top) and its Radon transform, $p(t, \theta)$ shown in a sinogram (bottom).

A sinogram has the X-ray angle θ and the detector position t , as its axes. Let's see what happens if we track a point in the cross-section.



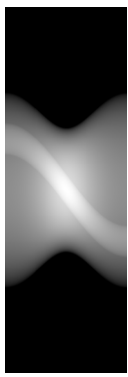
In the top cross-section view of Figure 7, we see a red spot. When an X-ray beam goes through this spot from different angles from 0 to 90 degrees, the projection corresponding to this red spot shows up in the sinogram, as shown in Figure 7 at the bottom. Its path follows a sine curve. This is where the term "sinogram" came from.



Why is this important? Sinograms are convenient representations of projection data, and they help us better understand what is happening in the projection space.

For example, if something goes wrong during CT data collection or a detector has hot or dead pixels, it is easy to spot them in a sinogram.

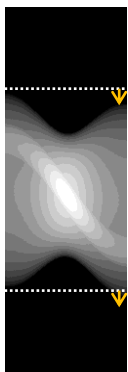
Figure 7: θ rotation in the cross-section and sinogram spaces



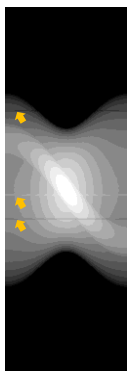
(a) Normal



(b) Vertical lines
They indicate either the detector or the X-ray source went dark during a scan.



(c) Shift with θ
It indicates the sample moved during a scan.



(d) Horizontal lines
They indicate the detector has hot, cold, or dead pixels.

A normal sinogram (Figure 8(a)) shows a collection of smooth sine curves.

Vertical lines darker or lighter than the surrounding areas, as shown in Figure 8(b), indicate the detector or the X-ray source went dark or bright during the scan.

As shown in Figure 8(c), an overall shift of the projection indicates the sample moved during the scan. When using the parallel beam geometry, the projection at 180 degrees should be a perfect mirror image of the one at 0 degree. In this example, the top and bottom edges shifted to the lower side at 180 degrees, indicating sample movement.

Horizontal lines that are darker or lighter than the surrounding areas, as shown in Figure 8(d), indicate that the detector has hot, cold, or dead pixels that response differently from the surrounding pixels.

As we will see in the hands-on exercises, these abnormalities cause various [artifacts](#) in the reconstructed CT images.

Figure 8: Various abnormalities identified in sinograms

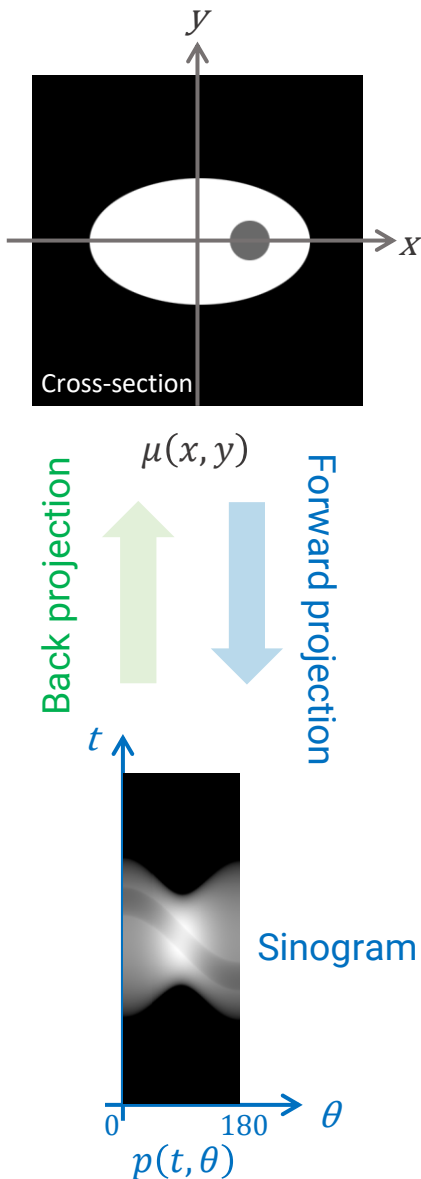
3

How does Filtered Back Projection work?

As we reviewed earlier, the Algebraic Reconstruction Technique reduces artifacts and requires a relatively small number of projections.

However, the calculation becomes complicated and starts taking a long time as the image matrix becomes large, a 3000 x 3000 x 3000 voxels cube, for example, a typical size of a micro-CT scan.

This is where the Filtered Back Projection (FBP) technique comes in handy. As the name suggests, it generates $\mu(x, y)$ by back-projecting $p(t, \theta)$.



Filtered Back Projection

In a CT data collection, we observe projections $p(t, \theta)$ and reconstruct the original absorption coefficient distribution $\mu(x, y)$.

The projection $p(t, \theta)$ is a forward projection or Radon transform of the original absorption coefficient distribution $\mu(x, y)$. In other words, $\mu(x, y)$ is a back projection of $p(t, \theta)$.

The Filtered Back Projection (FBP) calculates the back projection of $p(t, \theta)$ to reconstruct $\mu(x, y)$. Because we already know the forward projection follows equation (*), we can calculate the back projection based on this equation.

Figure 9: Forward and backward projections

Graphical Representation of FBP

We will review the FBP technique using a graphical representation using the egg-like object's cross-section we used in the previous section.

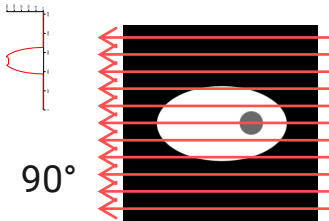
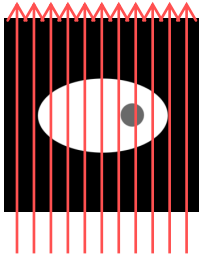
When X-rays go through the object in a 0-degree direction, we obtain a projection shown at the top of Figure 10. The gray "yolk" causes a dent in the projection due to its lower density than the surroundings. At 90 degrees, as shown at the bottom of Figure 10, the dent shows up in the center.

We can convert these projections into gray-level images and add them, as shown in Figure 11. The "dent" in each projection indicates where the "yolk" is. As you see in Figure 11, by adding these projections, we can eventually figure out where the "yolk" is.

Projection at 0°



0°



90°

Figure 10: Projections at 0 and 90 degrees.

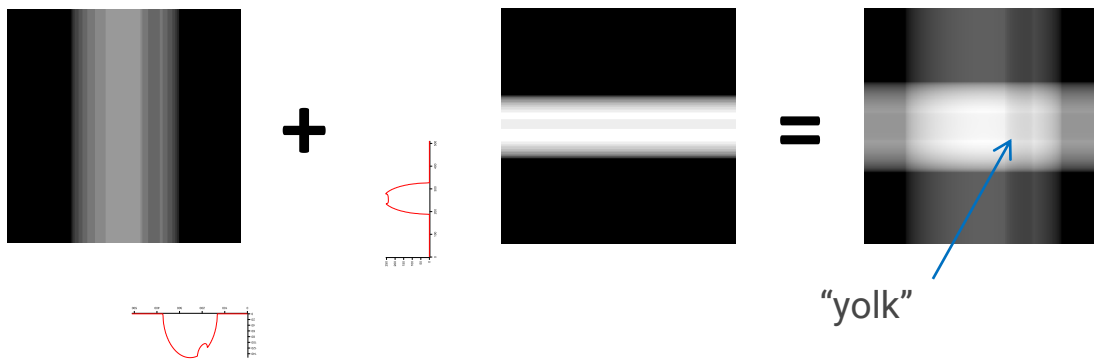
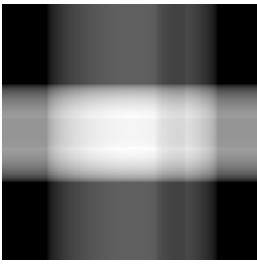
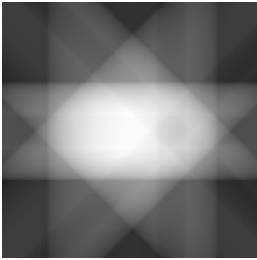


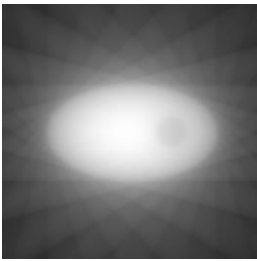
Figure 11: Adding projections from 0 and 90 degrees.



4 projections



8 projections



24 projections



120 projections

Figure 12:
Reconstructed cross-sections from 4, 8, 24, and 120 projections.

The reconstruction from two projections might not look like much, but you can already see the rough size of the “egg” and the location of the “yolk.”

As you increase the number of projections, the reconstructed cross-section becomes more refined, as shown in Figure 12.

In theory, the number of projections you need, N , to accurately reconstruct the original image is calculated as:

$$N = \frac{\text{Field of view}}{\text{voxel size}}$$

Recall the example of the Algebraic Reconstruction Technique. The field of view was two-pixel wide. So $N = 2$. We used 0 and 90 degrees, two projections, to solve four relational equations. That is the exact amount of information we need to reconstruct the original image.

Now, let’s consider why these images in Figure 12 are blurred.

Fourier Slice Theorem

To understand what blurs the reconstructed image, we need to go back to the math behind it.

We will skip the deriving of the relationship, but the 1D Fourier transform of the projection $p(t, \theta)$ is equal to a slice, at θ , of $F(u, v)$ that is a 2D Fourier transform of $\mu(x, y)$. See Figure 13.

This is called the Fourier slice theorem or the projection slice theorem. It is a convenient theory, as we will see next.

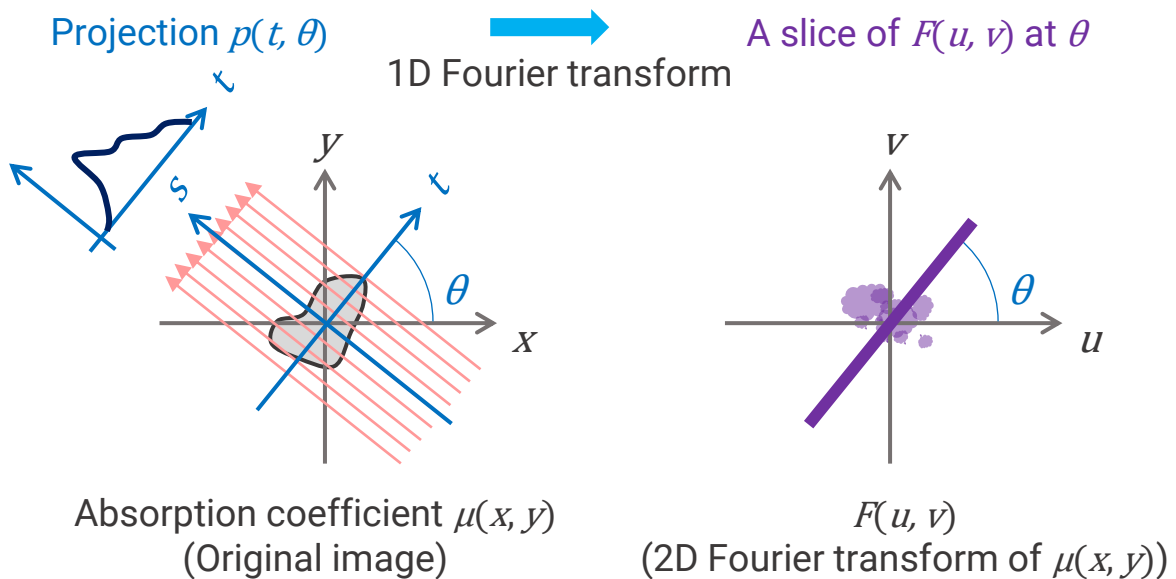
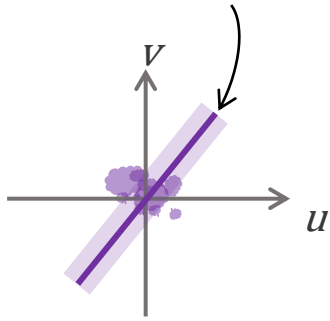


Figure 13: Relationship of the original image, its Fourier transform, and the projection

A line in theory
A rectangular in reality

A slice of $F(u, v)$ at θ



$F(u, v)$
(Fourier transform
of $\mu(x, y)$)

Figure 13: The theory and reality of a slice $F(u, v)$ at angle θ .

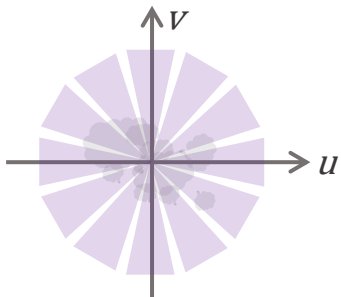
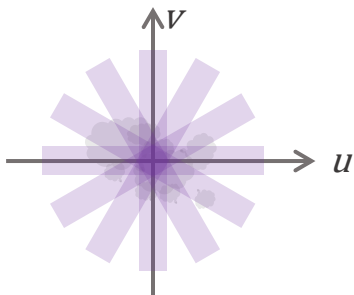


Figure 14: Without (top) and with (bottom) ramping filter

The Fourier slice theorem tells us all we need to do to reconstruct the original image is as follows:

1. Collect projections.
2. Fourier transform projections.
3. Add all projections in the Fourier space.
4. Inverse Fourier transform the results.

We need to consider that a slice of projection is not a line in reality but a rectangular shape with a width, as shown in Figure 13. This width causes uneven adding as shown in Figure 14 at the top. The center area has many data points added while the edge has fewer. This is what causes the blurring of the reconstructed image we saw on page 18.

To avoid this issue, we need to downweigh the center part of the slices, as shown at the bottom of Figure 14. The downweighing process is called filtering. Typical filters include the ramping, Hann, Ramachandran-Lakshminarayanan, and Shepp-Logan filters.

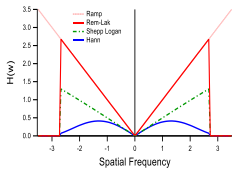


Figure 15: Various filters

Figure 15 shows some of the representative filters. As you will see in the hands-on exercise section, these filters can affect the resolution and contrast of the reconstructed images.

Further Reading

Toda, H. (2021). *X-ray CT: Hardware and Software Techniques*. Springer.

<https://doi.org/10.1007/978-981-16-0590-1>

If you are interested in how the Fourier slice theorem is derived, this video shows the process: [Intro to Digital Image Processing by Rich Radke - # 18 Reconstruction from parallel projections and the Radon transform.](#)

If you want to understand what exactly a Fourier transform is, this visual representation of the concept might help: [But what is the Fourier Transform? A visual introduction by 3Blue1Brown.](#)

5

ImageJ hands-on exercises

Hands-on exercises help us understand how the forward projection and the back projection, i.e., reconstruction, work.

In this section, we will apply forward and back projections to [sample images](#) and observe the effects of the number of projections, filters, and abnormalities in sinograms using an open-source [Fiji distribution of ImageJ](#) and [Radon transform plugin](#).

If you are new to ImageJ, watch this [Mini Tutorial: ImageJ Getting Started Guide](#).

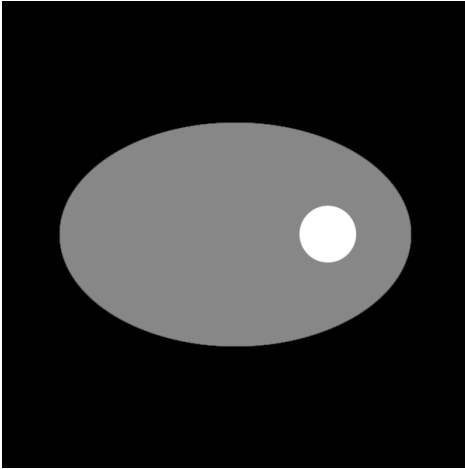
PLAY WITH THE SETTING PARAMETERS TO SEE HOW THE RESULTS CHANGE AS YOU GO THROUGH EXERCISES.

Go Deeper

ImageJ is a convenient tool for image processing and gives us a quick hands-on experience. But if your goal is to write your own code for image processing, [MATLAB](#) is also a good place to start. [Intro to Digital Image Processing](#) by Prof. Rich Radke is an excellent introductory course for image processing and uses MATLAB to demonstrate various filters and operations.

Radon Transformation

Let's generate 1D projections from a 2D cross-section using Radon transformation.

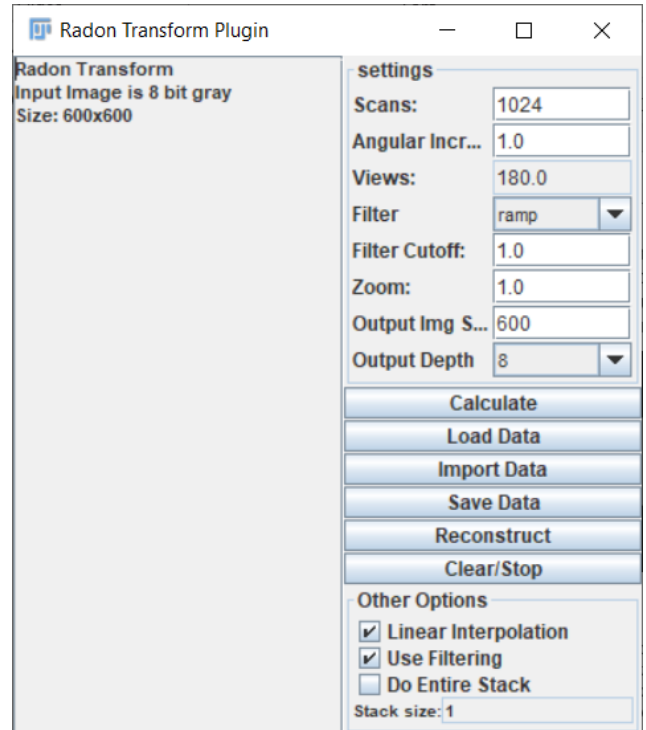


2D cross-section: An egg-like object. Black indicates the air with virtually no absorption. The gray egg and white yolk have medium and high absorption coefficients, respectively.

Generating sinograms

(Plugins Menu → Radon Transform)

We will use the Radon Transform plugin to generate a sinogram, which is a collection of projections.



The optimum number of **Scans** is automatically calculated by the plugin based on the size of the original cross-section.

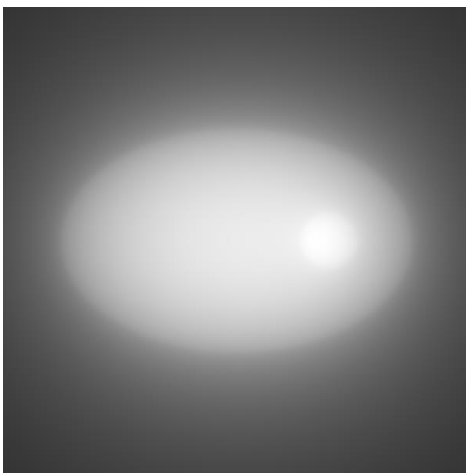
The **Angular increments** can be edited. This tool assumes the parallel beam geometry and calculates sinograms from 0 to 180 degrees. If you set the **Angular increments** to 1 degree, for example, the tool will generate 180 views (projections.)

Click the **Calculate** button to run the calculation.



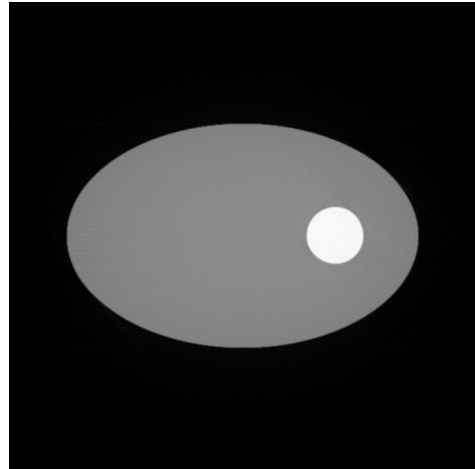
Sinogram: 180 projections with 1-degree increments

Now, let's reconstruct the original image using this sinogram. First, click the **Reconstruct** button and run the reconstruction with **"Use Filtering"** off.



Reconstruction: From 180 projections, without filter

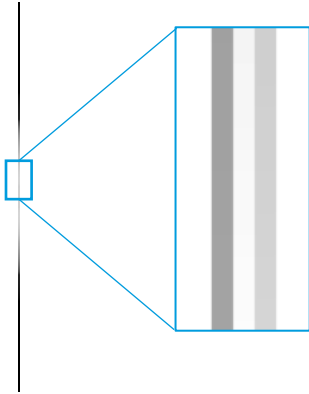
Compare the results with and without the filter.



Reconstruction: From 180 projections, with the ramp filter

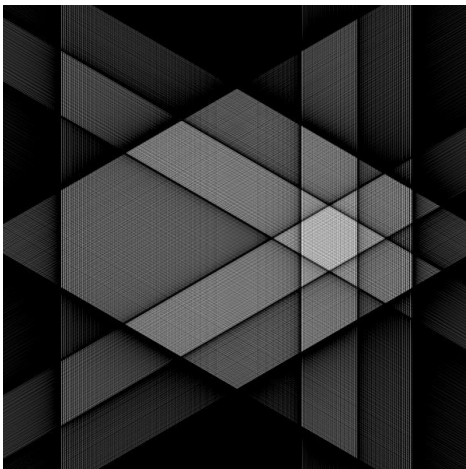
You can see the drastic change in the reconstruction results. The ramp filter eliminated the blurring.

Next, let's see what happens when we use very few projections. Change the Angular increments to 60 degrees and generate three projections.



Sinogram: 3 projections with 60-degree increments

Reconstruct the original image using this sinogram with the ramp filter.



Reconstruction: From 3 projections, with the ramp filter

As you see in this example, a lack of projections causes line-like artifacts and angular and not a smooth representation of the object in the reconstructed image.

Sinograms with abnormalities

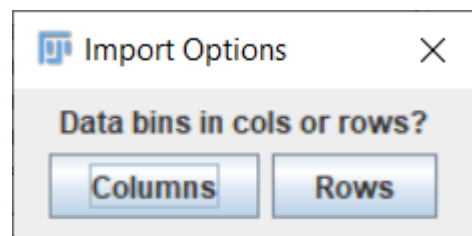
Let's look at sinograms with abnormalities caused by various problems that happened during data collection and see how they affect the reconstructed images.

Sinogram with vertical lines

Click the **Import Data** button and select a sinogram with a vertical line.

(egg_Sinogram_vertical_line.tif)

Click "Columns."

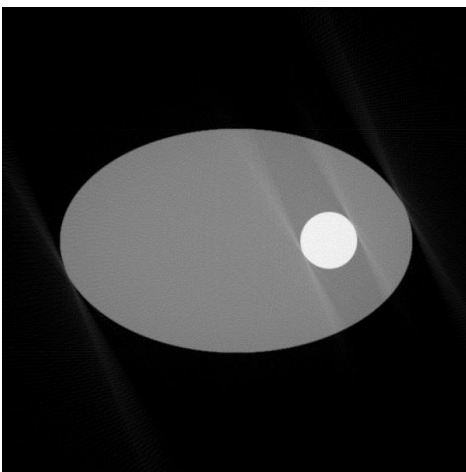


The dark vertical line in the sinogram indicates the detector or the X-ray source went dark during the data collection.



Sinogram: A dark line runs vertically.

Click the **Reconstruct** button to generate the reconstruction. (with the ramp filter)



Reconstruction: The dark vertical line caused tilted streaks.

You see streak artifacts at an angle where the detector or the source went dark.

Sinogram with distortion

Repeat the same process using the distorted sinogram.

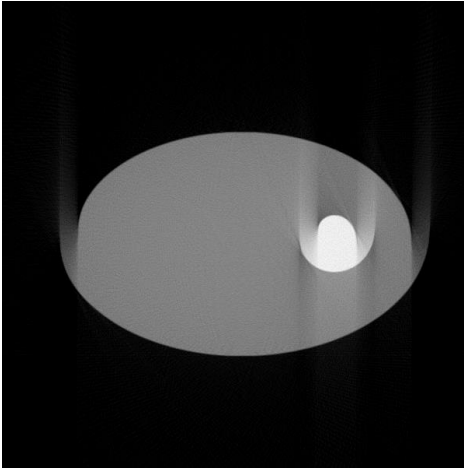
(egg_Sinogram_distorted.tif)

This type of distortion indicates that the sample moved or changed its shape during the data collection.



Sinogram: The bright part shifts downwards with increasing projection angle.

The resulting reconstruction exhibits streak artifacts.



Reconstruction: The distortion of the sinogram caused half-circle and line streaks.

The artifacts caused by sample movement often look similar to the artifacts caused by an incorrect focus correction. Not being able to find the correct focus value is a sign of sample movement. Or, you can investigate the sinogram to see if the sample moved because the sinogram is not affected by the focus correction.

Sinogram with horizontal lines

Repeat the same process using the sinogram with horizontal lines.

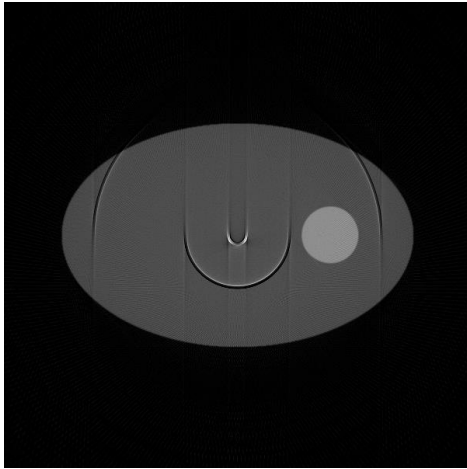
(egg_Sinogram_horizontal.tif)



Sinogram: Dark and light lines run horizontally.

These horizontal lines in the sinogram indicate the detector has hot, cold, or dead pixels with sensitivity different from the surrounding pixels.

The resulting reconstruction exhibits half-circle lines.

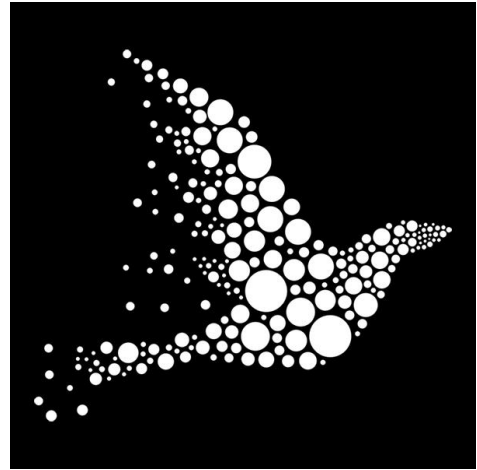


Reconstruction: The horizontal lines in the sinogram caused artifacts that look like half-circle lines.

When the sinogram runs from 0 to 360 degrees, as in the case of the cone beam geometry, the half-circle lines becomes full rings. This is how the commonly observed ring artifacts are caused.

Filters

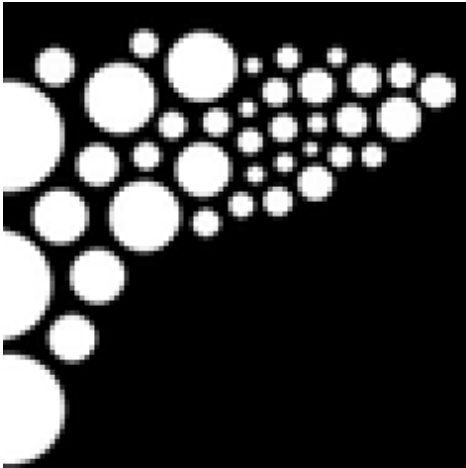
Let's compare different filters and see how they affect the reconstructed images.



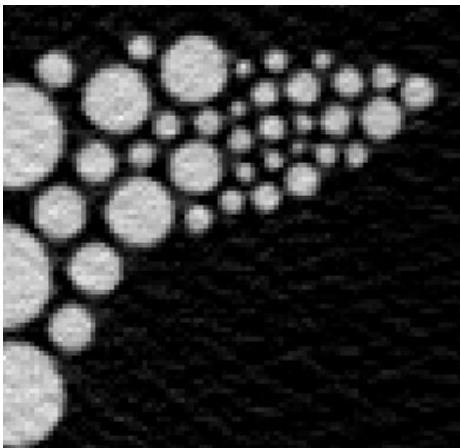
2D cross-section: bird.tiff

Calculate a Radon transform of this image and reconstruct it using different filters.

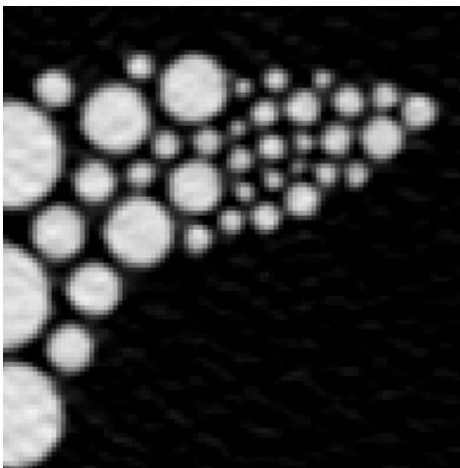
Original



Ramp filter



Hann filter



Although the reconstructed images are very close to the original, we can see some differences.

The ramp filter reconstruction has lower contrast and higher noise. The edges of the white circles look relatively sharp.

Meanwhile, the Hann filter reconstruction shows blurred interfaces between the white circles and black background. At the same time, it exhibits higher contrast and lower noise.

3D Volume Data

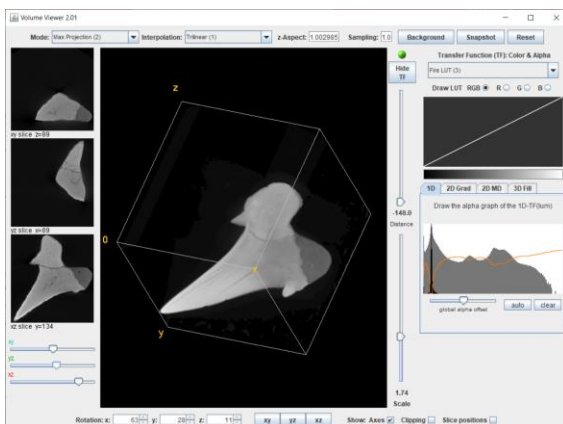
We have been using 2D cross-sections and their 1D projections. However, when we run CT scans, we scan a 3D volume and often observe its 2D projections.

Let's apply the Radon transform to 3D volume data to see how it works.

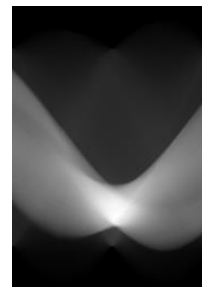
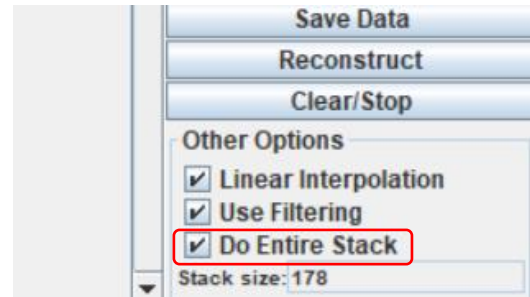
Open the 3D volume data of a shark tooth. (shark_tooth_180um.tiff)

Then display the 3D rendering of the data.

(Plugins Menu → Volume Viewer)



Run the Radon transform and generate a sinogram. Check “Do Entire Stack” to run it on the entire stack.



Sinogram: One of the 178 sinograms

178 sinograms are generated. The number 178 comes from the number of slices in the original image.

Reconstruct the 3D volume from these sinograms and confirm that the original object is restored.



ABOUT THE TOOLS

Fiji: A distribution of ImageJ

ImageJ is an open-source image processing program for scientific multidimensional images. We used Fiji distribution in this workshop. You can download Fiji for Windows, Mac, or Linux from this link:

<https://imagej.net/software/fiji/downloads>

Radon transform plugin

The Radon transform plugin used in this workshop is available from this link:

<https://imagej.nih.gov/ij/plugins/radon-transform.html>

Takeaways

Reconstruction is what makes CT possible. You might use software to run reconstruction and don't need to understand how it works. But when something goes wrong, understanding this theory is essential to realize that something is not right and to figure out how to fix the problem.

You might also face a need to reduce the number of projections, increase the scan speed without sacrificing the image quality, etc. Using a different reconstruction technique, such as the iterative or deep learning reconstruction, can satisfy these needs. Understanding the basic theory of reconstruction also helps you choose the right alternative reconstruction technique.

LET'S LEARN TOGETHER

Many people have learned what X-ray computed tomography (CT) is, how it works, and where it can be helpful in our webinar and workshop series. All recordings, application examples, a publication list, and blog articles are available at imaging.rigaku.com.

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